

Tech Brief

PUBLISHED BY THE NATIONAL ENVIRONMENTAL SERVICES CENTER

Fundamentals of Hydraulics: Flow

By **Zane Satterfield, P. E.**, NESC Engineering Scientist

Summary

Hydraulics is the branch of engineering that focuses on the practical problems of collecting, storing, measuring, transporting, controlling, and using water and other liquids. This *Tech Brief*—the second in a two-part series—provides basic information about hydraulic problems and will focus on calculating flow rates in water or wastewater conveyance (distribution and collection) systems. The first *Tech Brief* in this series discussed various aspects of pressure.

Why is understanding hydraulics important?

Liquid in motion produces forces and pressure whenever its velocity or flow direction changes. Knowing pipe pressure and flow at certain points along the pipe's path can help determine the necessary pipe size and capacity, as well as what pipe material would work best in given situations. Understanding hydraulics can help systems decide what pipe-flow rates related to size and material are necessary to transport water in an efficient manner. A key rationale for understanding hydraulics is that it can save money for small systems, especially those that can't afford engineering services or may not have an engineer on retainer.

Units and Variables

Understanding that water has a unit weight is important. System designers need this information to calculate other variables. The physical and hydraulic behavior of wastewater is similar to that of clean water, and there is generally no difference in the design or analysis of systems involving these two liquids. Both are considered to be incompressible liquids because their volume does not change significantly with changing pressure.

Water or wastewater has a unit weight of 62.4 pounds per cubic foot (62.4 lb/ft³); so, one cubic foot (1ft x 1ft x 1ft) of water weighs 62.4 pounds. There are 7.48 gallons in a cubic foot, with each gallon weighing approximately 8.34 pounds, so 7.48 gal x 8.34 lbs/gal = 62.4 pounds.

In the metric system, the term for weight (force due to gravity) is a Newton (N), and the unit weight of water is 9,800 Newtons per cubic meter (9,800 N/m³). More appropriately, this is expressed as 9.8 kilonewtons per cubic meter (9.8 kN/m³), where the prefix kilo stands for 1,000.

Flow

In pipes under pressure or in open channels under the force of gravity, the volume of water flowing past any given point in the pipe or channel per unit time is called the flow rate or discharge (Q).

In the U.S., flow rate may be expressed as cubic feet per second (ft³/s or cfs), gallons per minute (GPM), or million gallons per day (MGD). An approximate but convenient conversion to remember is 1 MGD = 1.55 cfs = 700 GPM.

In the metric system, the unit for flow rate is cubic meters per second (m³/s). The term liters per second (L/s) is also used for relatively small flow rates. Another expression for flow rate is megaliters per day (ML/d), so 1 m³ = 1000 L and 1 ML = 10⁶ L.

Flow rate is expressed by the formula:

$$Q = A \times V$$

Where Q = flow rate or discharge in gpm, mgd or cfs (ft³/s) in U. S. — m³/s, ML/d or L/s in SI metric units

A = cross-sectional flow area usually in ft² in U. S. units — m² in SI metric units

V = velocity of flow usually in ft/s in U. S. units — m/s in SI metric units

Flow in Pipes under Pressure

When water flows in a pipe, friction occurs between the flowing water and the pipe wall, and between the layers of water moving at different velocities in the pipe due to the viscosity of the water. The flow velocity is zero at the pipe wall and maximum along the centerline of the pipe. Velocity of flow means the average velocity over the cross section of the flow.

The frictional resistance to flow causes energy loss in the system. This energy loss is a continuous pressure

drop along the path of flow.

To be able to design new water distribution pipelines or sewage force mains or to analyze existing pipe networks, you will need to calculate the head losses, pressures, and flows throughout the system. There are several formulas in hydraulics to do this, but one of those most commonly used is the Hazen-Williams equation:

$$Q = 0.28 \times C \times D^{2.63} \times S^{0.54}$$

Where Q = flow rate in m^3/s in SI metric or gpm in U. S.

0.28 = constant which can be used for both SI units and U.S. units with sufficient accuracy.

C = pipe roughness coefficient (dimensionless) can be found in tables based on pipe material such as Table 1.

D = pipe diameter in (m) SI units or (in) inches U.S. units

S = slope of hydraulic grade line, dimensionless

$$S = h_L/L$$

Where h_L is head loss in feet or meters

Where L is the length of pipe in feet or meters

Example 1

Table 1. Hazen-Williams Coefficient for Different Types of Pipe

Pipe Materials	C_{HW}
Asbestos Cement	140
Brass	130-140
Brick Sewer	100
Cast Iron	
New, unlined	130
10 yr. old	107-113
20 yr. old	89-100
30 yr. old	75-90
40 yr. old	64-83
Concrete or concrete lined	
Steel forms	140
Wooden forms	120
Centrifugally spun	135
Copper	130-140
Galvanized Iron	120
Glass	140
Lead	130-140
Plastic	140-150
Steel	
Coal-tar enamel lined	145-150
New unlined	140-150
Riveted	110
Tin	130
Vitrified Clay (good condition)	110-140
Wood Stave (average condition)	120

Your water system services a vacant industrial park with existing water lines in place. A company wants to use the entire industrial park site and needs 1,800 gpm. The existing 12-inch diameter new, plastic pipe carries water with a head loss of 10 feet per 1,000 feet of pipeline. Determine the flow rate in the pipe using the Hazen-Williams equation to see if the existing piping will handle their demands giving that the storage and pressure head is adequate.

Solution: From Table 1, the Hazen-Williams equation finds the pipe roughness coefficient (C) for a new, plastic pipe to be $C = 140$.

Now compute the value of S , where $S = h_L/L$

$$S = 10 \text{ ft}/1,000 \text{ ft}$$

$$S = 0.010$$

The diameter is given in inches and stays in inches

$$D = 12 \text{ in}$$

Now applying the Hazen-Williams equation

$$Q = 0.28 \times C \times D^{2.63} \times S^{0.54}$$

$$Q = 0.28 \times 140 \times 12^{2.63} \times 0.010^{0.54}$$

Most calculators have a y^x button. If yours does not, get one that does.

$$Q = 0.28 \times 140 \times 689.04 \times 0.08317$$

$$Q = 2,246 \text{ gpm}$$

The calculation shows that the existing 12-inch pipe will provide more than the required amount needed by the new facility.

Gravity Flow in Pipes or Open Channels (Not under Pressure)

When water flows in a pipe or a channel with a free surface exposed to the atmosphere, it is called open channel or gravity flow. Gravity provides the moving force. But, as with pressure flow, there are friction losses or energy losses in gravity flow as well. Stream or river flow is open channel flow. Flow in storm or sanitary sewers is also open channel flow, except when the water is pumped through the pipe under pressure (a force main).

Most routine calculations in the design or analysis of storm or sanitary sewer systems involve a condition called steady uniform flow. Steady flow means that the discharge is constant with time. Uniform flow means that the slope of the water surface and cross-sectional flow area are also constant in the length of channel being analyzed.

Under steady, uniform-flow conditions, the slope of the water surface is the slope of the channel bottom.

The top of the inside pipe wall is called the crown, and the bottom of the pipe wall is called the invert. One basic objective of sewer design is to establish appropriate invert elevations along the pipeline. The length of wetted surface on the pipe or stream cross section is called the wetted perimeter. The size of the channel or pipe, as well as its slope and wetted perimeter, are important factors in rela-

tion to its discharge capacity.

The common formula for solving open channel flow problems is called Manning's formula and is written as:

$$Q = 1.0 \text{ or } 1.5/n \times A \times R^{2/3} \times S^{1/2}$$

Where Q = channel discharge capacity in (m³/s) SI units or (ft³/s) U.S. units

1.0 = constant for SI metric units

1.5 = constant for U.S. units

n = channel roughness coefficient (Manning's n) dimensionless

A = cross-sectional *flow* area (not the cross-section of pipe or channel) in m² SI units or ft² U.S. units

R = hydraulic radius of the channel in (m) SI units or (ft) U.S. units

S = slope of the channel bottom, dimensionless

From the variables above the hydraulic radius of a channel R , is defined as the ratio of the cross-sectional flow area A to the wetted perimeter P . In formula form:

$$R = A/P$$

Where R = hydraulic radius of the channel in (m) SI units or (ft) U.S. units

A = cross-sectional *flow* area (not the cross-section of pipe or channel) in m² SI units or ft² U.S. units

P = wetted perimeter in (m) SI units or ft U.S. units

The roughness coefficient (n), often referred to as Manning's n , depends on the material and age of the pipe or lined channel and on topographic features for a natural streambed. It can range from a value of 0.01 for a smooth, clay pipe to 0.1 for a small natural stream. A value of n commonly assumed for concrete pipes or concrete lined channels is 0.013.

Example 2

Type of Pipe	Manning's n	
	Min.	Max.
Glass, brass, or copper	0.009	0.013
Smooth cement surface	0.010	0.013
Wood-stave	0.010	0.013
Vitrified sewer pipe	0.010	0.017
Cast-Iron	0.011	0.015
Concrete, precast	0.011	0.015
Cement mortar surfaces	0.011	0.015
Common-clay drainage tile	0.011	0.017
Wrought Iron	0.012	0.017
Brick with cement mortar	0.012	0.017
Riveted-steel	0.017	0.020
Cement rubble surfaces	0.017	0.030
Corrugated metal storm drain	0.020	0.024

A storm sewer pipe is going to be extended to an existing 3-foot-wide by 2-foot-deep concrete lined channel. The extended storm sewer pipe has a peak discharge of 20 cfs (ft³/s). A 6-inch freeboard—the difference between the top of the water and the container it is in—is needed in the channel. Will the channel handle the flow with 6-inch freeboard?

A rectangular drainage channel is 3-foot wide and is lined with concrete. The bottom of the channel drops in elevation at a rate of 0.5 feet per 100 feet. What is the discharge from the channel when the water depth is flowing at 1.5 feet that gives us a 6-inch freeboard? Use Manning's n of 0.013 for a concrete-lined channel.

Solution: Because the data is in U.S. units, use the constant of 1.5 in the Manning's equation.

The cross-sectional flow area (A) in the channel is:

$$A = 3.0 \text{ ft wide} \times 1.5 \text{ ft water depth}$$

$$A = 4.5 \text{ ft}^2$$

The wetted perimeter P is:

$$P = 1.5 \text{ ft (side)} \times 3.0 \text{ ft (bottom)} \times 1.5 \text{ ft (side)}$$

$$P = 6.0 \text{ ft}$$

The hydraulic radius R is:

$$R = A/P$$

$$R = 4.5 \text{ ft}^2 / 6.0 \text{ ft}$$

$$R = 0.75 \text{ ft}$$

The slope S is:

$$S = \text{rise (drop)}/\text{run}$$

$$S = 0.5 \text{ ft} / 100 \text{ ft}$$

$$S = 0.005$$

Using Manning's equation we can calculate Q discharge:

$$Q = (1.5/n) \times A \times R^{2/3} \times S^{1/2} \text{ or } (0.5)$$

It is very helpful if your calculator has a y^x button

$$Q = (1.5/0.013) \times 4.5 \text{ ft}^2 \times 0.75^{2/3} \text{ ft} \times 0.005^{1/2}$$

$$Q = 115.38 \times 4.5 \text{ ft}^2 \times 0.8255 \text{ ft} \times 0.07071$$

$$Q = 30.3 \text{ cfs (ft}^3/\text{s)}$$

The concrete lined channel will handle the extended storm sewer with a peak discharge of 20 cfs (ft³/s) with plenty of freeboard.

Circular Pipes Flowing Full

Most sanitary and storm sewer systems are built with circular pipe. Along with the roughness coefficient, the other important factors in design or analysis are the flow velocity (V), pipe diameter (D), pipe slope (S), and how much the pipe can handle flow full (max discharge Q).

In a circular pipe carrying water so that the pipe is just full to the crown (but still under atmospheric pressure and gravity flow), the flow area (A) would be the pipe cross-sectional area ($\pi D^2/4$), the area of the pipe $A = (\pi \times D^2)/4$. The wetted perimeter (P) is the perimeter of the pipe, or circle or $P = \pi \times D$. Because the hydraulic radius (R) is defined as A divided by P, ($R = A/P$), we get:

$$R = A/P$$

$$R = ((\pi \times D^2)/4) / (\pi \times D)$$

Or written as

$$R = \frac{(\pi \times D^2)}{4} \times \frac{1}{(\pi \times D)}$$

This can be reduced to:

$$R = D/4$$

For circular pipes flowing full, the Manning's formula is:

$$Q = ((1.0 \text{ or } 1.5)/n) \times ((\pi \times D^2)/4) \times (D/4)^{2/3} \times S^{1/2} \text{ or } 0.5$$

For a given value n, only the pipe diameter and slope are needed to solve for discharge (Q) in a circular pipe flowing full.

To help in analysis there are charts or nomographs for the Manning's equation just as with the Hazen-Williams equation.

Example 3

A combined sewer overflow is calculated to be 14 cfs (ft³/s). The existing discharge point needs to be redirected under a road. A 24-inch corrugated metal pipe is on hand in the supply yard. Will this pipe work?

Calculate the discharge of a 24-inch corrugated metal pipe flowing full at a slope of two percent.

From Table 2, use the Manning's roughness coefficient n. The table has a range for corrugated metal storm drain from 0.020 min for new condition to 0.024 for older condition. Because the condition is not given, use the middle range of 0.022, or to generate a worst-case condition, use 0.024. However, this example uses the middle range of 0.022, $n = 0.022$.

The diameter is given at 24 inches but needs to be in feet; there are 12 inches in a foot so, 24 in / 12 in per foot = 2 ft (D = 2 ft).

The slope is given at two percent. To get this into decimal form, 2% / 100 = 0.02. (S = 0.02).

In U.S. units, use the constant of 1.5 in Man-

ning's equation for a full flowing pipe:

$$Q = (1.5/n) \times (\pi \times D^2)/4 \times (D/4)^{2/3} \times S^{1/2} \text{ or } 0.5$$

$$Q = (1.5/0.022) \times (\pi \times 2^2)/4 \times (2/4)^{2/3} \times 0.02^{1/2}$$

$$Q = 68.18 \times 3.1415 \times 0.6299 \times 0.1414$$

$$Q = 19.08 \text{ cfs (ft}^3/\text{s)}$$

The combined sewer overflow is 14 cfs (ft³/s). This equation illustrates that the calculated flow for the 24-inch corrugated metal pipe flowing full at two-percent slope will handle 19.08 cfs (ft³/s). The existing 24-inch pipe will work.

Closing

Hydraulics can be complex when designing or analyzing entire collection or distribution systems. Knowing the correct formulas to calculate forces or pressure is the first step in having a properly functioning system. Pipe size, tank storage, and location are all key in getting water to system customers. If any situation should come up that the water system staff does not feel comfortable analyzing, they should consult a licensed engineer.

Several software and spreadsheet programs are available to assist in the hydraulics of system design. However, the data they generate is only as good as the information that is input. Knowing the fundamentals of hydraulics can help users get the most out of these programs.

References

- Hwang, Ned H. C. and Robert J. Houghtalen. 1996. *Fundamentals of Hydraulic Engineering Systems*, Third Edition. New Jersey: Prentice-Hall, Inc.
- Lindeburg, Michael R. 1999. *Civil Engineering Reference Manual for the PE Exam*. Seventh Edition. Belmont, CA: Professional Publications, Inc.
- Nathanson, Jerry A. 1997. *Basic Environmental Technology, Water Supply, Waste Management and Pollution Control*. Second Edition. New Jersey: Prentice-Hall, Inc.



NESC Engineering Scientist **Zane Satterfield** is a licensed professional engineer and water operator who previously worked for the West Virginia Bureau of Public Health, Environmental Engineering Division.